

SPHALERONS WITH TWO HIGGS DOUBLETS *

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We report on work studying the properties of the sphaleron in models of the electroweak interactions with two Higgs doublets in as model-independent a way as possible: by exploring the physical parameter space described by the masses and mixing angles of the Higgs particles. If one of the Higgs particles is heavy, there can be several sphaleron solutions, distinguished by their properties under parity and the behaviour of the Higgs field at the origin. In general, these solutions are not spherically symmetric, although the departure from spherical symmetry is small.
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1 Introduction

One of the major unsolved problems in particle cosmology is accounting for the baryon asymmetry of the Universe. This asymmetry is usually expressed in terms of the parameter η , defined as the ratio between the baryon number density n_B and the entropy density s : $\eta = n_B/s \sim 10^{-10}$. Sakharov ¹ laid down the framework for any explanation: the theory of baryogenesis must contain B violation; C and CP violation; and a departure from thermal equilibrium. All these conditions are met by the Standard Model ² and its extensions, and so there is considerable optimism that the origin of the baryon asymmetry can be found in physics accessible at current and planned accelerators (see ³ for reviews).

Current attention is focused on the Minimal Supersymmetric Standard Model, where there are many sources of CP violation over and above the CKM matrix ⁴, and the phase transition can be first order for Higgs masses up to 120 GeV, providing the right-handed stop is very light and the left-handed stop very massive ⁵.

B violation is provided by sphalerons ⁶, at a rate $\Gamma_s \simeq \exp(-E_s(T)/T)$, where $E_s(T)$ is the energy of the sphaleron at temperature T . This rate must not be so large that the baryon asymmetry is removed behind the bubble wall, and this condition can be translated into a lower bound on the sphaleron mass

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$E_s(T_c)/T_c \gtrsim 45$. Thus it is clear that successful baryogenesis requires a careful calculation of the sphaleron properties.

Here we report on work on sphalerons in the two-doublet Higgs model (2DHM) in which we study the properties of sphalerons in as general a set of realistic models as possible. In doing so we try to express parameter space in terms of physical quantities: Higgs masses and mixing angles, which helps us avoid regions of parameter space which have already been ruled out by LEP.

Previous work on sphalerons in 2DHMs^{7,8,9,10} has restricted either the Higgs potential or the ansatz in some way. Our potential is restricted only by a softly broken discrete symmetry imposed to minimize flavour-changing neutral currents (FCNCs). Our ansatz is the most general spherically symmetric one, including possible C and P violating field configurations¹¹.

We firstly check our results against the existing literature, principally BTT⁸, who found a new P-violating “relative winding” (RW) sphaleron, specific to multi-doublet models, albeit at $M_A = M_{H^\pm} = 0$. This is distinguished from Yaffe’s P-violating deformed sphaleron¹¹ or “bisphaleron” by a difference in the behaviour of each of the two Higgs fields at the origin. We then reexamine the sphaleron in a more realistic part of parameter space, where M_A and M_{H^\pm} are above their experimental bounds. We reiterate the point made in¹² that introducing Higgs sector CP violation makes a significant difference to the sphaleron mass (between ten and fifteen percent), and may significantly change bounds on the Higgs mass from electroweak baryogenesis.

2 Two Higgs doublet electroweak theory

The most general quartic potential for 2DHMs has 14 parameters, only one of which, the Higgs vacuum expectation value, v , is known. However, we are aided by the observation¹³ that FCNCs can be suppressed by imposing a softly-broken discrete symmetry $\phi_{1(2)} \rightarrow +(-)\phi_{1(2)}$, and results in a potential with 10 real parameters. One of these may be removed by a phase redefinition of the fields; the vacuum configuration is then entirely real, and CP violation is contained in one term $2\chi_2 \left(\text{Re}(\phi_1^\dagger \phi_2) - \frac{v_1 v_2}{2} \right) \text{Im}(\phi_1^\dagger \phi_2)$. Ignoring couplings to other fields, when $\chi_2 = 0$ there is a discrete symmetry $\phi_\alpha \rightarrow -i\sigma_2 \phi_\alpha^*$, which sends $\text{Im}(\phi_1^\dagger \phi_2) \rightarrow -\text{Im}(\phi_1^\dagger \phi_2)$. This can be identified as C invariance.

Following¹² we determine as many as possible of the nine parameters in the potential from physical ones. The physical parameters at hand are the four masses of the Higgs particles, the three mixing angles of the neutral Higgses, one of which, θ_{CP} , is the only CP violating physical parameter, (θ_{CP} mixes the CP even and CP odd neutral Higgs sector), and v .

We further check that the potential for these sets of physical parameters is always bounded from below.

3 Sphaleron ansatz and numerical methods

The most general static spherically symmetric ansatz is, in the radial gauge

$$\phi_\alpha = (F_\alpha + iG_\alpha \hat{x}^a \sigma^a) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad W_i^a = \left[\frac{(1+\beta)}{r} \varepsilon_{aim} \hat{x}_m + \frac{\alpha}{r} (\delta_{ai} - \hat{x}_a \hat{x}_i) \right]. \quad (1)$$

Here, the subscript $\alpha = 1, 2$, and $F_\alpha = a_\alpha + ib_\alpha$ and $G_\alpha = c_\alpha + id_\alpha$ are complex functions. The boundary conditions can be most easily expressed in terms of the functions χ , Ψ , H_α , h_α , and Θ_α , defined by

$$-\beta + i\alpha = \chi e^{i\Psi}, \quad a_\alpha + ic_\alpha = H_\alpha e^{i\Theta_\alpha}, \quad b_\alpha + id_\alpha = h_\alpha e^{i\Theta_\alpha}, \quad (2)$$

and one can show that as $r \rightarrow 0$, either $H_\alpha^2 + h_\alpha^2 \rightarrow 0$ or $\Theta_1 \rightarrow \Psi/2 + n_1\pi$ and $\Theta_2 \rightarrow \Psi/2 + n_2\pi$, ($n_1, n_2 \in \mathbf{Z}$). These boundary conditions distinguish between the various types of sphaleron solution: the ordinary sphaleron has $H_\alpha^2 + h_\alpha^2 \rightarrow 0$ as $r \rightarrow 0$, the bisphaleron has non-vanishing Higgs fields with $n_1 = n_2$, and the RW sphaleron non-vanishing Higgs fields with $n_1 \neq n_2$. These integers represent the winding of the Higgs field around spheres of constant $|\phi_\alpha|$, although only their difference has any gauge invariant meaning.

Note that the ansatz is potentially inconsistent, as $\text{Im}(\phi_1^\dagger \phi_2) \propto \hat{x}_3$, a point which has not been noted before. In practice, the non-spherically symmetric parts of the static energy functional, $E[f_A]$, contribute less than 1% of the total, and so we are justified in assuming the fields of the ansatz, f_A , are a function only of the radial co-ordinate r and then integrating over \hat{x}_3 .

We look for solutions to $E[f_A]$ using a Newton method¹¹ which is an efficient way of finding extrema (and not just minima). The method can be briefly characterised as updating the fields f_A by δf_A , given by the solution of $E'' \delta f = -E'$, where the primes denote functional differentiation with respect to f_A . A particular advantage to using this method is that because we calculate E'' , it is straightforward to get the curvature eigenvalues, and therefore to check that the solution really is the lowest energy unstable solution.

4 Results

We first checked our method and code against the results of BTT⁸, and Yaffe¹¹ finding agreement in the energy of better than 1 part in 10^3 for a wide range of parameters. We also measured the Chern-Simons numbers, n_{CS} , of the solutions that they discovered and determined that they were near 1/2,

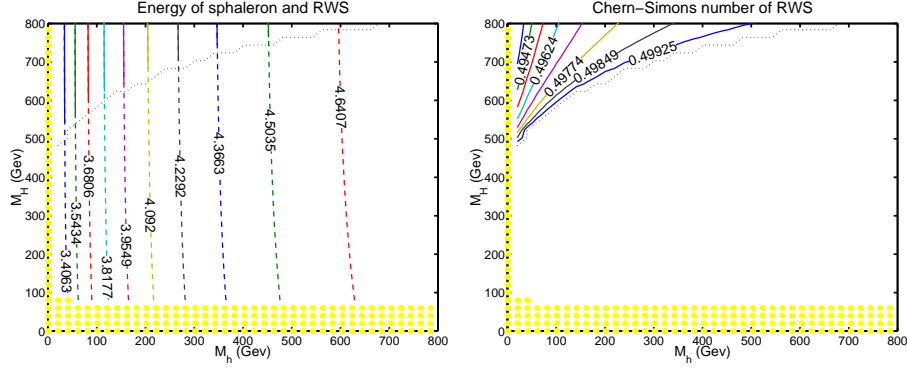


Figure 1. Contours of sphaleron energy (M_W/α_W) and Chern-Simons number.

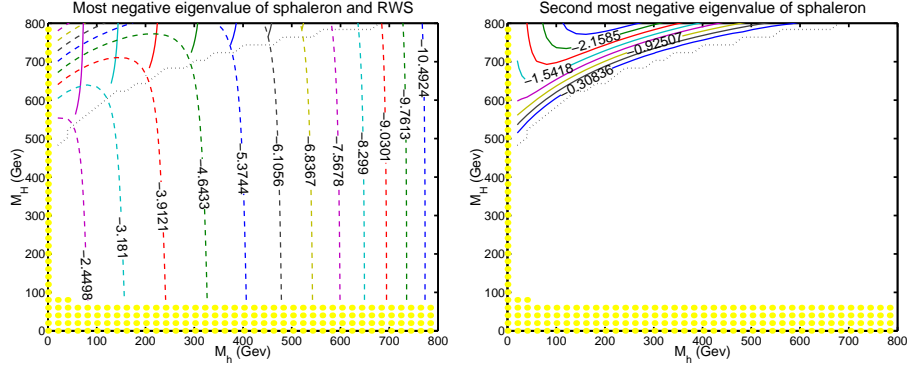


Figure 2. Eigenvalues (M_w) of the sphaleron solutions as a function of the CP even Higgs masses M_h and M_H . There was no mixing, and $M_A = 241$ GeV, $M_{H^\pm} = 161$ GeV, $\tan \beta = 6$, $\lambda_3 = -0.05$. The solid lines in the most negative eigenvalue plot represent the relative winding sphaleron, while the dashed lines are for the ordinary sphaleron, for the dotted region the potential was unbounded from below.

but not exactly $1/2$ as with the sphaleron solution. Further they appeared in P conjugate pairs with n_{CS} of the pair adding to exactly one. We then looked at more realistic values of M_A and M_{H^\pm} , with results that are displayed in Figs. 1 and 2. Note first of all the well-known feature that the sphaleron mass depends mainly on M_h . Secondly, for increasing M_H , the curvature matrix of the sphaleron develops a second negative eigenvalue (Fig. 2, right), signalling

the appearance of a pair of RW sphalerons. The lower of the two n_{CS} is plotted on the right Fig. 1. The departure from $n_{CS} = 1/2$ is small, as is the difference in energy between the RW and ordinary sphalerons for the Higgs masses we examined, however the most negative curvature eigenvalue of the RW sphaleron can be double that of the sphaleron. More detailed results and discussion of their significance are reserved for a future publication ¹⁴.

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